Calculus

IB SL Study Guide

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IB Math AI SL — Calculus

Complete Study Guide

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Topic 5 of the IB Math AI SL syllabus — Paper 1 and Paper 2

IB TIP

Applications focus: In Math AI, calculus is about solving real problems — finding maximum profit, minimum cost, total distance, accumulated quantity. You need to know the rules, but the emphasis is on setting up and interpreting the model, not algebraic manipulation. Your GDC handles much of the computation.

MEMORISE THIS

Core differentiation rules

Function	Derivative
x^n	nx^{n-1}
ax^n	anx^{n-1}
e^x	e^x
$\ln x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
Constant c0	

Section 1: Differentiation

1.1 What is a Derivative?

The **derivative** $f'(x)$ (or $\frac{dy}{dx}$) gives the **instantaneous rate of change** of a function at any point. Geometrically, it is the **gradient of the tangent line** at that point.

1.2 The Power Rule

For $f(x) = ax^n$:

$f'(x) = anx^{n-1}$

This works for any real exponent n, including negative and fractional values.

WORKED EXAMPLE

Power rule — basic differentiation

Find the derivative of $f(x) = 3x^4 - 5x^2 + 7x - 2$.

$f'(x) = 12x^3 - 10x + 7$

Each term is differentiated separately. The constant -2 disappears.

1.3 Derivatives of Exponential and Trigonometric Functions

$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$

EXAM ALERT

Radians only for trig calculus: When differentiating or integrating trigonometric functions, your GDC must be in **radian mode**. The formulas above only work in radians.

1.4 Interpreting the Derivative in Context

If $f(t)$ represents a quantity over time, then $f'(t)$ represents the **rate of change** of that quantity.

Context	$f(t)$	$f'(t)$
Population over time	Number of individuals	Growth rate
Profit vs. units sold	Total profit	Marginal profit
Distance vs. time	Position	Velocity
Velocity vs. time	Velocity	Acceleration

WORKED EXAMPLE

Derivative in context — population growth

The population of a city (in thousands) is modelled by $P(t) = 50e^{0.03t}$ *where* t *is years after 2020. Find the rate of population growth in 2030.*

$P'(t) = 50 \times 0.03 \times e^{0.03t} = 1.5e^{0.03t}$

At $t = 10$ (year 2030):

$P'(10) = 1.5e^{0.3} = 1.5 \times 1.3499 = 2.02$ thousand people per year.

The population is growing at approximately 2020 people per year in 2030.

Section 2: Applications of Differentiation

2.1 Finding Stationary Points

A **stationary point** (also called a turning point or critical point) occurs where $f'(x) = 0$.

To classify a stationary point:

Method 1 — **Second derivative test:**

- If $f''(x) > 0$, it is a **local minimum**
- If $f''(x) < 0$, it is a **local maximum**
- If $f''(x) = 0$, the test is inconclusive

Method 2 — **Sign of $f'(x)$:** Check the gradient on either side of the stationary point.

2.2 Optimization

Optimization means finding the maximum or minimum value of a quantity. This is the most important application of differentiation in Math AI.

Strategy:

1. Identify the quantity to optimize (profit, area, cost, etc.)
2. Write it as a function of one variable
3. Differentiate and set $f'(x) = 0$
4. Solve for x
5. Verify it is a maximum or minimum
6. Calculate the optimal value

WORKED EXAMPLE

Optimization — maximum revenue

A company sells widgets. If the price is p dollars, the number sold per week is $n = 800 - 20p$*. Revenue* $R = p \times n$*. Find the price that maximizes revenue.*

$R(p) = p(800 - 20p) = 800p - 20p^2$

$R'(p) = 800 - 40p$

Set $R'(p) = 0$: $800 - 40p = 0 \implies p = 20$

$R''(p) = -40 < 0$, confirming a maximum.

Maximum revenue: $R(20) = 800(20) - 20(400) = 16000 - 8000 = 8000$ dollars per week.

At a price of 20 dollars per widget, revenue is maximized at 8000 dollars per week, selling $800 - 20(20) = 400$ widgets.

WORKED EXAMPLE

Optimization — minimum material

An open-top box is made from a square piece of card by cutting squares of side x *cm from each corner and folding up. The original card is 24 cm by 24 cm. Find the value of* x *that maximizes the volume.*

After cutting, the base is $(24 - 2x)$ by $(24 - 2x)$ and the height is x.

$V(x) = x(24 - 2x)^2 = x(576 - 96x + 4x^2) = 4x^3 - 96x^2 + 576x$

$V'(x) = 12x^2 - 192x + 576$

Set $V'(x) = 0$: $12x^2 - 192x + 576 = 0 \implies x^2 - 16x + 48 = 0$

$(x - 4)(x - 12) = 0 \implies x = 4$ or $x = 12$.

Since $x = 12$ would mean no box (cutting the entire card), $x = 4$ cm.

$V(4) = 4(16)^2 = 1024$ cm^3.

2.3 Equation of the Tangent Line

The tangent to $y = f(x)$ at $x = a$ has equation:

$y - f(a) = f'(a)(x - a)$

✎ **WORKED EXAMPLE**

Tangent line

Find the equation of the tangent to $y = x^3 - 2x + 1$ *at* $x = 2$.

$y(2) = 8 - 4 + 1 = 5$. Point: $(2, 5)$.

$y' = 3x^2 - 2$. At $x = 2$: $y'(2) = 12 - 2 = 10$.

Tangent: $y - 5 = 10(x - 2) \implies y = 10x - 15$.

2.4 Kinematics (Motion)

If $s(t)$ is position:

- **Velocity:** $v(t) = s'(t) = \frac{ds}{dt}$
- **Acceleration:** $a(t) = v'(t) = s''(t) = \frac{d^2s}{dt^2}$

✎ **WORKED EXAMPLE**

Kinematics — particle motion

A particle moves along a line with position $s(t) = t^3 - 6t^2 + 9t + 2$ *metres, where* $t \geq 0$ *seconds.*

(a) Find the velocity at $t = 1$.

$v(t) = 3t^2 - 12t + 9$

$v(1) = 3 - 12 + 9 = 0$ m/s. The particle is momentarily at rest.

(b) Find when the particle changes direction.

The particle changes direction when $v(t) = 0$:

$3t^2 - 12t + 9 = 0 \implies t^2 - 4t + 3 = 0 \implies (t - 1)(t - 3) = 0$

$t = 1$ and $t = 3$ seconds.

(c) Find the acceleration at $t = 2$.

$a(t) = 6t - 12$

$a(2) = 12 - 12 = 0$ m/s^2.

Section 3: Integration

3.1 Antiderivatives (Indefinite Integrals)

Integration is the reverse of differentiation.

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$

$\int e^x \, dx = e^x + C$

$\int \frac{1}{x} \, dx = \ln|x| + C$

The constant C is the **constant of integration**, needed because many different functions have the same derivative.

✎ WORKED EXAMPLE

Indefinite integral

Find $\int (6x^2 - 4x + 3) \, dx$.

$= \frac{6x^3}{3} - \frac{4x^2}{2} + 3x + C = 2x^3 - 2x^2 + 3x + C$

3.2 Definite Integrals (Area Under a Curve)

$\int_a^b f(x) \, dx = F(b) - F(a)$

where $F(x)$ is any antiderivative of $f(x)$.

Geometric interpretation: The definite integral gives the **signed area** between the curve and the x-axis from $x = a$ to $x = b$.

- Area above the x-axis is **positive**
- Area below the x-axis is **negative**

For **total area** (always positive), take the absolute value of each region.

✎ WORKED EXAMPLE

Definite integral — total distance

A particle has velocity $v(t) = 3t^2 - 12$ m/s. Find the total distance travelled from $t = 0$ to $t = 3$.

First find where $v(t) = 0$: $3t^2 = 12 \implies t = 2$.

For $0 \leq t < 2$: $v(t) < 0$ (moving in the negative direction). For $2 < t \leq 3$: $v(t) > 0$ (moving in the positive direction).

Total distance $= \left| \int_0^2 (3t^2 - 12) \, dt \right| + \int_2^3 (3t^2 - 12) \, dt$

$\int_0^2 (3t^2 - 12) \, dt = [t^3 - 12t]_0^2 = (8 - 24) - 0 = -16$

$\int_2^3 (3t^2 - 12) \, dt = [t^3 - 12t]_2^3 = (27 - 36) - (8 - 24) = -9 + 16 = 7$

Total distance $= |-16| + 7 = 16 + 7 = 23$ m.

EXAM ALERT

Displacement vs. distance: $\int_a^b v(t)\,dt$ gives **displacement** (net change in position, can be negative). For **total distance**, you must split the integral where $v(t) = 0$ and take absolute values. This distinction is tested regularly.

3.3 Finding Area Between Curves

$A = \int_a^b |f(x) - g(x)|\,dx$

On the GDC, graph both functions and use the intersection points as limits.

Section 4: The Trapezoidal Rule

When a function cannot be integrated analytically, use the **trapezoidal rule** for numerical approximation.

$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$

where $h = \dfrac{b-a}{n}$ is the width of each strip and n is the number of strips.

MEMORISE THIS

Trapezoidal rule — simplified

Area $\approx \frac{h}{2}\,[\text{first} + \text{last} + 2 \times (\text{sum of all middle values})]$

WORKED EXAMPLE

Trapezoidal rule — river width

A river's cross-section is measured at 5 equally spaced points. The depths (m) are:

Distance from bank (m)	0	3	6	9	12
Depth (m)	0	1.8	2.4	1.5	0

Estimate the cross-sectional area.

Here $h = 3, n = 4$ strips.

$A \approx \frac{3}{2}[0 + 0 + 2(1.8 + 2.4 + 1.5)] = \frac{3}{2}[0 + 2(5.7)] = \frac{3}{2}(11.4) = 17.1\text{ m}^2$

4.1 Accuracy of the Trapezoidal Rule

- More strips (larger n) gives a better approximation
- The rule **overestimates** when the curve is concave up
- The rule **underestimates** when the curve is concave down
- It is exact for linear functions

💡 **IB TIP**

When to use the trapezoidal rule: Use it when you have a table of values (from data or experiment) and no algebraic formula, or when the function is too complex to integrate. The IB gives the trapezoidal rule in the formula booklet.

Section 5: Practice Questions

Paper 1 Style (Short Answer)

▶ **Q1.** Find the derivative of $f(x) = 5x^3 - 2x^2 + 4$.
▶ **Q2.** Find $\int(4x^3 + 6x - 1)\,dx$.
▶ **Q3.** The profit function is $P(x) = -2x^2 + 120x - 500$ where x is the number of units sold. Find the number of units that maximizes profit.

Paper 2 Style (Extended Response)

▶ **Q4.** The rate of water flowing into a tank is $R(t) = 20 - 4t$ litres per minute, where t is time in minutes. (a) Find the volume of water that flows in during the first 3 minutes. (b) At what time does the flow stop? (c) Find the total volume that enters the tank.
▶ **Q5.** A cylindrical can must hold 500 ml of liquid. (a) Show that the surface area is $A = 2\pi r^2 + \frac{1000}{r}$. (b) Find the radius that minimizes the surface area. (c) Find the minimum surface area.
▶ **Q6.** The speed of a car (m/s) is recorded every 2 seconds as it accelerates from rest.

⚠ **EXAM ALERT**

Optimization questions always follow the same pattern: define the function, differentiate, set equal to zero, solve, verify max/min, give the answer in context. Practise setting up the function from a word problem — this is the step most students find hardest.

November 2026 Prediction Questions

⚠ **EXAM ALERT**

These are NOT official IB questions. These are trend-based practice questions written to reflect the topic areas and question styles most likely to appear on the November 2026 IB Math AI SL Paper 2. Based on recent exam patterns (2022–2025), expect heavy weighting on: real-world optimization (maximize revenue or minimize cost), integration to find total accumulated quantity in context, and the trapezoidal rule applied to a data table.

WORKED EXAMPLE

Question 1 — Optimization: Maximize Revenue [~8 marks]

A market stall sells handmade candles. The stall owner finds that when the price is set at p dollars, the number of candles sold per day is $n(p) = 120 - 8p$.

(a) Write an expression for the daily revenue $R(p)$ in terms of p.

(b) Find $R'(p)$.

(c) Find the price that maximizes daily revenue, and verify it is a maximum.

(d) Find the maximum daily revenue.

(e) The owner's daily cost is $C = 2.50 \times n + 45$ dollars (cost per candle plus fixed overheads). Find the price that maximizes daily profit, and find that profit.

▶ Show Solution

WORKED EXAMPLE

Question 2 — Integration to Find Total Quantity in Context [~7 marks]

Water drains from a reservoir. The rate of outflow (in megalitres per hour) at time t hours after draining begins is modelled by:

$$R(t) = 12e^{-0.3t}, \quad 0 \leq t \leq 10$$

(a) Find the rate of outflow at $t = 0$ and interpret this value.

(b) Find the total volume of water that drains from the reservoir in the first 4 hours.

(c) Find the total volume of water that drains over the full 10 hours.

(d) Find the time at which the rate of outflow has fallen to half its initial value.

▶ Show Solution

WORKED EXAMPLE

Question 3 — Trapezoidal Rule with a Real Dataset [~6 marks]

A scientist measures the concentration C (mg/L) of a drug in a patient's bloodstream at regular time intervals after administration.

Time t (hours)	0	1	2	3	4	5	6
Concentration C (mg/L)	0	4.8	7.2	6.9	5.1	3.2	1.8

(a) Use the trapezoidal rule with all seven data points to estimate $\int_0^6 C\,dt$.

(b) State the units of your answer and explain what this value represents in context.

(c) The effective therapeutic window requires the total drug exposure (the area under the curve) to be between 25 and 40 mg·h/L. Determine whether this dose falls within the therapeutic window.

(d) State whether the trapezoidal rule overestimates or underestimates the true area between $t = 0$ and $t = 2$. Give a reason.

▶ Show Solution